Standard deviation and the Student's T-test

Help sheet for determining if two means are statistically different

1. Imagine Ms. Franzen germinated 50 beans; 25 in distilled water, and 25 in salt water. The beans in distilled water took 1.0 day to germinate and the beans in salt water took 1.2 days to germinate on average. Do these results mean that beans watered with salt water take longer to germinate? Or… was the small difference due to chance variation only? How do we know?

2. In order to determine whether or not your two samples are statistically different and that the difference is caused by the treatment instead of by chance, we use a test called the student's t-test.

3. Before you begin your calculations, you must write a null hypothesis (H_0) and an alternative hypothesis (H_A)
   - H_0: The differences in the sets of data are the result of chance variation only and there <u>is no</u> treatment effect.
   - H_A: The differences in the sets of data are statistically significant and there <u>is</u> a treatment effect.

4. In order to perform the student's t-test, several steps must be followed:
   a. First, calculate the mean (average) of each of the two experimental groups.
   b. Next calculate the standard deviation of each of the two data groups. Standard deviation is a measure of the "scatter" of the data. {See attached sheet for a better description of standard deviation} The larger the "scatter" of the data, the more likely any difference is due to change; the tighter the data, the more likely the difference is due to the treatment of the two groups.
   c. The equation for standard deviation is shown below:

$$\sigma = \sqrt{\frac{\Sigma(X - \mu)^2}{N}}$$

ex:

$$\sqrt{\frac{(1-2)^2 + (2-2)^2 + (3-2)^2}{3}} = \sqrt{.667} = .817$$

- μ = the mean
- N = the number of trials
- x = each experimental value

This formula calculates the deviation of each experimental value in a group from its group's mean.

17th Annual Health & Hygiene Drive

Student Poster Contest

April 5 is the kick-off date for the 17th Annual Health & Hygiene Drive. Milwaukee Public Schools, Local 1053, and United Way of Greater Milwaukee are sponsors of this event. Each year, many necessary health and hygiene items are gathered through the generous staff and students of Milwaukee Public Schools. To many of us, having these items may be taken for granted. However, to those individuals who stay in shelters, having them is very much appreciated. Each year, those items gathered are donated to over 25 shelters located throughout Milwaukee.

In order to raise the level of awareness relative to the great need the drive addresses, and to increase participation in the drive, all schools are invited to participate in the Health and Hygiene Drive Student Poster Contest. Details are as follows:

Participation: ***The contest is open to all MPS students attending schools that are participating in the 17th Annual Health & Hygiene Drive. Those schools wishing to participate in the art contest and/or the 17th Annual Health & Hygiene Drive must complete the attached form and return it to Susan Saller, Office of the Superintendent, room 235, before March 6, 2006.***

Dates of Contest: ***March 6 – March 24.***

Final Submissions: ***Completed entries should be dropped off in the Office of the Superintendent by Friday, March 24. Judging will take place immediately so posters can be returned to schools for display that coincides with the dates of the drive (April 5 – May 26). *A completed submission form (attached) must be clipped to each entry.***

Objective of Contest: ***Raise the level of awareness about the need the drive addresses and promote participation in the drive.***

Judging Criteria: ***Central Services administrators will judge submitted posters. Judges will be looking for creative and colorful visual graphics paired with written slogans (optional) that illustrate the need the drive addresses and encourages participation in the drive. Approximate size of submitted posters: 22" x 17".***

Awards: ***First, second, and third place awards will be presented for each of the divisions. Each winner will receive a deluxe plaque engraved with their name and their school's name.***

Divisions: ***Kindergarten; 1st – 2nd grade; 3rd – 5th grade; 6th – 8th grade; 9th – 12th grade.***

d. Once you have the mean of each group and the standard deviation of each group, you can calculate the t -value.

e. Calculate the t-value using the following equation:

$$t = \frac{\left|\bar{X}_1 - \bar{X}_2\right|}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}$$

- X_1= the mean of group 1
- X_2= the mean of group 2
- S_1= the calculated standard deviation of group 1
- S_2= the calculated standard deviation of group 2
- N_1= the number of trials in group 1
- N_2= the number of trials in group 2

f. Before you use the t-table, you must calculate the degrees of freedom. This can be done by adding the number of trials in each group and subtracting two (E.X. (25+25) - 2= 48 = degrees of freedom

g. Once you get your t-value and you know your degrees of freedom, you can compare it to a student's t-table to determine whether you can support or refute your null hypothesis (whether the difference in your two means is due to the treatment or to chance)

h. Using the attached chart, find your degrees of freedom and the confidence interval you wish to use. (We will use p=0.05 which we will use to prove with 95% confidence that are results are due to chance OR the treatment) Circle the number that is found where this column and row meet. This is your Critical value.

i. If your calculated t-value is GREATER than your critical value, you will reject H_0 and accept H_A. If the calculated t-value is LESS than your critical value, you will accept H_0.

j. You may wish to say in your conclusion "The value of the t-statistic (3.06) well exceeds the critical value for P=0.05 at 10 degrees of freedom. As a result we can reject H_0 and conclude that the means are different with 95% confidence." ...or vice versa.

SAMPLE LETTER FOR PARENTS/GUARDIANS

*Please feel free to personalize the content of this letter for your school community. The information could also be adjusted for publication in a school newsletter.

March 2006

Dear Parent/Guardian,

Our school community is participating in the district's 17th Annual Health & Hygiene Drive. Each year, many necessary health and hygiene items are gathered through generous MPS families and staff, and donated to over 25 shelters located throughout Milwaukee.

The official dates of the drive are April 5 – May 26. Your son or daughter may be asking you if your family would like to donate some items to the drive. Please consider participating by picking up an extra item or two at the grocery store, or donating sealed items that your family will not be using.

Suggested items include soap, toothbrushes/toothpaste, mouthwash, combs/brushes, lip balm, paper products, shaving cream, hair care products, baby products, laundry supplies, deodorant, feminine products, and first aid supplies. Hotel/travel size items are especially helpful to the shelters.

Thank you in advance for supporting the 17th Annual Health & Hygiene Drive!

Sincerely,

variation in populations

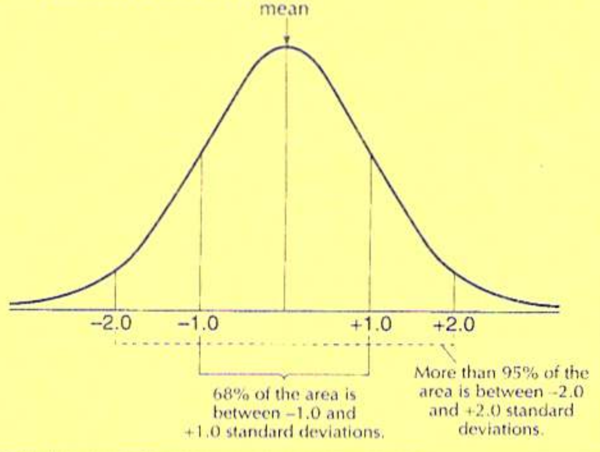
Although members of a population show similarities because they are members of the same species, they also show differences – **variation**. For example, humans vary in height and in skin colour.

The range of variation can be shown using a graph called a frequency distribution. Most variation gives a bell-shaped frequency distribution called the **normal distribution**.

The mean value is in the middle of the distribution. Another statistic called the **standard deviation** is used to assess how far the values are spread above and below the mean. A high standard deviation shows that the data is widely spread and a low standard deviation shows that the data are clustered closely around the mean. A useful rule is that 68% of the values lie between one standard deviation above and below the mean in a normal distribution (right).

The standard deviation can be used to help decide whether the difference between two means is likely to be significant. Two examples are described below.

The normal distribution



Variation in Holly berries

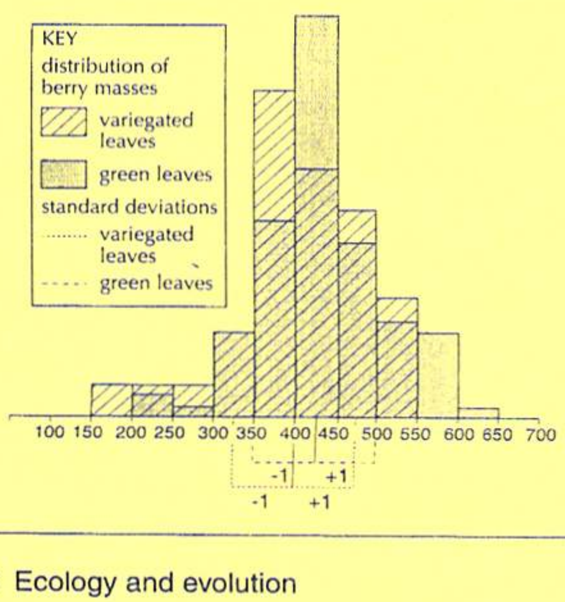
A group of students collected holly berries from two *Ilex aquifolium* trees. One tree had green leaves and the other variegated leaves (leaves that were partly green and partly yellow).

Hypothesis: the berries from the tree with variegated leaves will be smaller than the berries from the tree with all-green leaves.

The mass of some berries from each tree was found. The mean mass and the standard deviation were calculated for each tree.

Tree	Mean berry mass	Standard deviation
Green leaves	427 mg	73 mg
Variegated leaves	399 mg	80 mg

The berries from the tree with green leaves had a 28 mg larger mean mass than those from the tree with variegated leaves. However the standard deviations (73 mg and 80 mg) are much larger than the difference between the means. The difference in the mean mass of the berries is therefore unlikely to be significant.



Variation in Bank voles

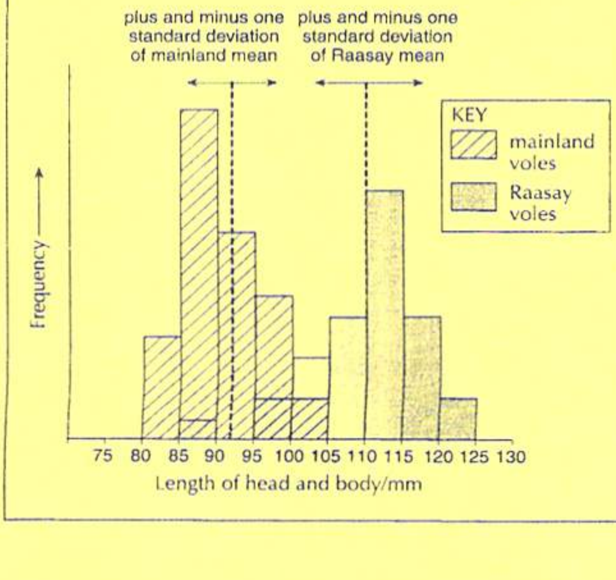
Ecologists noticed that bank voles (*Clethrionomys glareolus*) seemed to grow to a larger size on Raasay, a small Scottish island than on the mainland.

Hypothesis: adult bank voles are larger on Raasay than on mainland Britain.

Adult voles were caught using small mammal traps on Raasay and on the mainland. The length of each vole was measured and the mean lengths and standard deviations were calculated.

Vole population	Mean length	Standard deviation
Mainland Britain	92 mm	5.2 mm
Raasay	110 mm	7.1 mm

The mean length of the bank voles on Raasay is 18 mm greater than mean length of those on the mainland. The standard deviations (5.2 and 7.1) are much smaller than the difference in the means. The difference in the length of the bank voles was therefore almost certainly significant – the population on Raasay grew to a larger size.



df	0.10	0.05	0.025	0.01
2	2.9200	4.3027	6.2054	9.9250
3	2.3534	3.1824	4.1765	5.8408
4	2.1318	2.7765	3.4954	4.6041
5	2.0150	2.5706	3.1634	4.0321
6	1.9432	2.4469	2.9687	3.7074
7	1.8946	2.3646	2.8412	3.4995
8	1.8595	2.3060	2.7515	3.3554
9	1.8331	2.2622	2.6850	3.2498
10	1.8125	2.2281	2.6338	3.1693
11	1.7959	2.2010	2.5931	3.1058
12	1.7823	2.1788	2.5600	3.0545
13	1.7709	2.1604	2.5326	3.0123
14	1.7613	2.1448	2.5096	2.9768
15	1.7531	2.1315	2.4899	2.9467
16	1.7459	2.1199	2.4729	2.9208
17	1.7396	2.1098	2.4581	2.8982
18	1.7341	2.1009	2.4450	2.8784
19	1.7291	2.0930	2.4334	2.8609
20	1.7247	2.0860	2.4231	2.8453
21	1.7207	2.0796	2.4138	2.8314
22	1.7171	2.0739	2.4055	2.8188
23	1.7139	2.0687	2.3979	2.8073
24	1.7109	2.0639	2.3910	2.7970
25	1.7081	2.0595	2.3846	2.7874
26	1.7056	2.0555	2.3788	2.7787
27	1.7033	2.0518	2.3734	2.7707
28	1.7011	2.0484	2.3685	2.7633
29	1.6991	2.0452	2.3638	2.7564
30	1.6973	2.0423	2.3596	2.7500
31	1.6955	2.0395	2.3556	2.7440
32	1.6939	2.0369	2.3518	2.7385
33	1.6924	2.0345	2.3483	2.7333
34	1.6909	2.0322	2.3451	2.7284
35	1.6896	2.0301	2.3420	2.7238
36	1.6883	2.0281	2.3391	2.7195
37	1.6871	2.0262	2.3363	2.7154
38	1.6860	2.0244	2.3337	2.7116
39	1.6849	2.0227	2.3313	2.7079
40	1.6839	2.0211	2.3289	2.7045
41	1.6829	2.0195	2.3267	2.7012
42	1.6820	2.0181	2.3246	2.6981
43	1.6811	2.0167	2.3226	2.6951
44	1.6802	2.0154	2.3207	2.6923
45	1.6794	2.0141	2.3189	2.6896
46	1.6787	2.0129	2.3172	2.6870
47	1.6779	2.0117	2.3155	2.6846
48	1.6772	2.0106	2.3139	2.6822
49	1.6766	2.0096	2.3124	2.6800
50	1.6759	2.0086	2.3109	2.6778
51	1.6753	2.0076	2.3095	2.6757
52	1.6747	2.0066	2.3082	2.6737
53	1.6741	2.0057	2.3069	2.6718
54	1.6736	2.0049	2.3056	2.6700
55	1.6730	2.0040	2.3044	2.6682
56	1.6725	2.0032	2.3033	2.6665
57	1.6720	2.0025	2.3022	2.6649
58	1.6716	2.0017	2.3011	2.6633
59	1.6711	2.0010	2.3000	2.6618
60	1.6706	2.0003	2.2990	2.6603
61	1.6702	1.9996	2.2981	2.6589
62	1.6698	1.9990	2.2971	2.6575
63	1.6694	1.9983	2.2962	2.6561
64	1.6690	1.9977	2.2954	2.6549
65	1.6686	1.9971	2.2945	2.6536
66	1.6683	1.9966	2.2937	2.6524
67	1.6679	1.9960	2.2929	2.6512
68	1.6676	1.9955	2.2921	2.6501
69	1.6672	1.9949	2.2914	2.6490
70	1.6669	1.9944	2.2906	2.6479
71	1.6666	1.9939	2.2899	2.6469
72	1.6663	1.9935	2.2892	2.6458
73	1.6660	1.9930	2.2886	2.6449
74	1.6657	1.9925	2.2879	2.6439
75	1.6654	1.9921	2.2873	2.6430
76	1.6652	1.9917	2.2867	2.6421
77	1.6649	1.9913	2.2861	2.6412
78	1.6646	1.9908	2.2855	2.6403
79	1.6644	1.9905	2.2849	2.6395
80	1.6641	1.9901	2.2844	2.6387
81	1.6639	1.9897	2.2838	2.6379
82	1.6636	1.9893	2.2833	2.6371
83	1.6634	1.9890	2.2828	2.6364
84	1.6632	1.9886	2.2823	2.6356
85	1.6630	1.9883	2.2818	2.6349
86	1.6628	1.9879	2.2813	2.6342
87	1.6626	1.9876	2.2809	2.6335
88	1.6624	1.9873	2.2804	2.6329
89	1.6622	1.9870	2.2800	2.6322
90	1.6620	1.9867	2.2795	2.6316
91	1.6618	1.9864	2.2791	2.6309
92	1.6616	1.9861	2.2787	2.6303
93	1.6614	1.9858	2.2783	2.6297
94	1.6612	1.9855	2.2779	2.6291
95	1.6611	1.9852	2.2775	2.6286
96	1.6609	1.9850	2.2771	2.6280
97	1.6607	1.9847	2.2767	2.6275
98	1.6606	1.9845	2.2764	2.6269
99	1.6604	1.9842	2.2760	2.6264
100	1.6602	1.9840	2.2757	2.6259

water: .5, 2, .5, 2, 1 $\mu = 1$

Salt water: 1.1, 1.3, 1.2, 1.2 $\mu = 1.2$

water $\sigma = \sqrt{\dfrac{(.5-1)^2+(2-1)^2+(.5-1)^2+(2-1)^2+(1-1)^2}{5}}$

$\sigma = \sqrt{\dfrac{(.25)+(1)+(.25)+(1)+(0)}{5}}$

$\sigma = \sqrt{\dfrac{2.5}{5}} = .707$

Bean Lab….

Due before class on Wednesday, March 15th.

Be certain you have your…

- Title
- Problem (Question)
- Hypothesis (If, then, because…)
- Procedure (STEP BY STEP)
- Data (Rough hand written form AND final charted form)
- Analysis (preliminary analysis should include averages for germination time for each different experimental or control group. This can be included on your final data chart)

…written out (or typed) in final form!

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IB Biology II – Bean Germination Lab

Objective: You are to design a lab that tests one (or more) factor affecting the germination of the Northern Bean (Phaseolus vulgaris).

Any materials you need should be requested prior to the start of your lab. Plan for your lab to be completed at your School over the course of two or more weeks. Please allow me to check over your procedure before you begin. Consider how many trials you will need; consider your experimental and control groups. Do not forget to use references to support / refute your data and conclusions. Reference these in a bibliography.